RESEARCH NOTE

Study of sub-critical Taylor vortex flow between eccentric rotating cylinders by torque measurements and visual observations

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This study of the flow between a rotating inner and stationary outer cylinder revealed secondary vortex flows at Taylor numbers well below the value at which Taylor vortices are formed. A distinct sequence of transitional stages has been observed. Torque measurements, visual observations, and varying the eccentricity have been used to study sub-critical vortices with zero axial flow

Keywords: *vortices, Taylor number, sub-critical, eccentricity*

Since the earlier work of Taylor[1], it has been known that the flow between a stationary outer cylinder and a rotating inner cylinder becomes unstable when a critical value of a non-dimensional parameter, Taylor number, is exceeded. This leads to the formation of a system of contra rotating, toroidal vortices, known as Taylor vortices.

Although there have been many references in the literature to the possible existence of secondary flows at sub-critical Taylor numbers, it is only recently that detailed theoretical and experimental investigations have been made. The early work contains a number of hints. Donnelly[2] observed a change in the slope of his torque–Taylor number curve just before the Taylor vortices became apparent visually while Schwarz, Springett, and Donnelly[3] obtained uncharacteristically low critical Taylor numbers. Synder and Lambert[4] observed that evidence has frequently appeared which might indicate the existence of another type of instability near the critical Taylor numbers. Vohr's visual observations[5] gave consistently low values of critical Taylor numbers compared with the values deduced from torque measurements. The torque measurements of Younes, Mobbs and Coney[6] indicated that secondary flows may sometimes be sufficiently strong to cause appreciable additions to the frictional torque.

Using dye injection, Castle and Mobbs[7] discovered two distinct instabilities at widely separated Taylor numbers for the case of eccentric cylinders. They observed the formation of an initial vortex system which was replaced by a second system with a considerably reduced axial wavelength at a higher Taylor number.

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At $\eta = 0.908$ and a length/gap ratio of 65, Jackson, Robati and Mobbs[8] detected weak vortex motions between concentric cylinders at 0.3 T_c using aluminium paint pigment for flow visualization. Using improved visualization techniques, Jackson and Mobbs[9] observed a distinctive sequence of secondary flows prior to critical Taylor vortex formation.

Recently Preston and Mobbs[10] considered flow between rotating inner and stationary outer cylinders with stationary end plates determining the annulus length. Their numerical solutions of the equations of motion subject to these boundary conditions show that a vortex structure is always present at any Taylor number. The strongest vortex cells lie adjacent to the end plates and the cells become weaker towards the centre. From this analysis, the authors concluded that the existence of secondary flows at sub-critical Taylor numbers is due to the end effects. This conclusion is supported by Walton[11] who considered the Taylor instability problem with boundary conditions applied at the annulus ends.

Apparatus

Two experimental rigs, both with a rotating inner and stationary outer cylinders, were used. The torque measurement rig, described in detail elsewhere[12], has a 600 mm long inner cylinder, 53.95 mm radius and a 59.99 mm radius outer cylinder, giving a radius ratio of 0.9. The rig has facilities for both inner and outer cylinder torque measurements and the outer cylinder can be moved eccentric to the inner cylinder.

The visual observation rig has been described by Younes *et al*[6]. The rig has an inner cylinder of 50.80 mm radius, a 57.15 mm radius perspex outer cylinder, giving a radius ratio of 0.889, and is 1219 mm long.

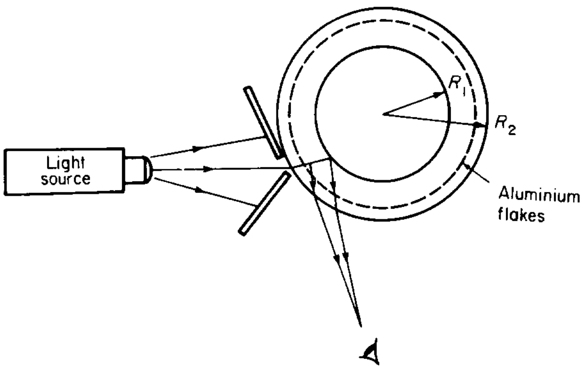

Fig 1 Visualization arrangement

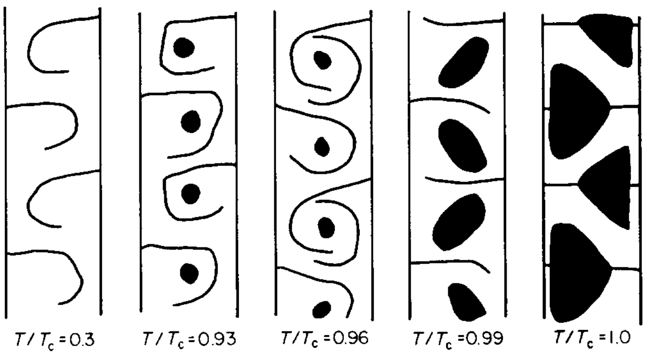

Fig 2 Development of vortices at sub-critical Taylor numbers

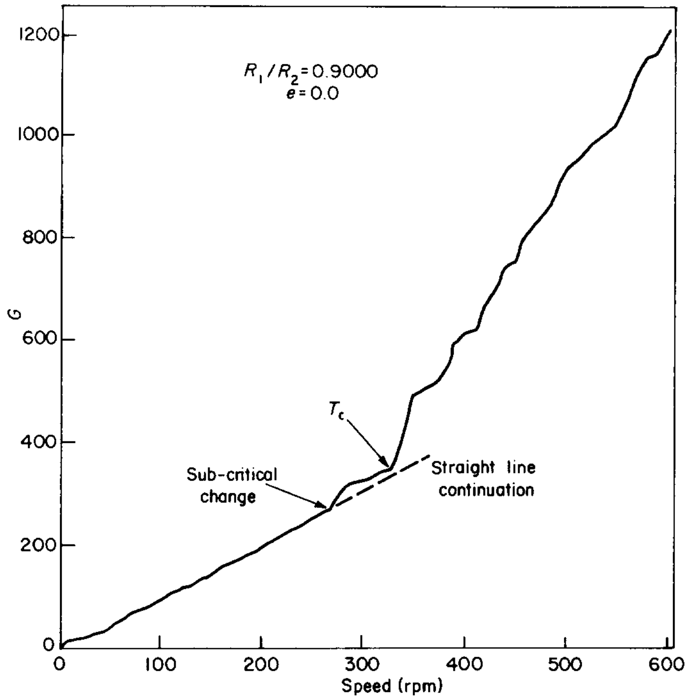

Fig 3 Effect of sub-critical vortices on torque, concentric case

Results and discussion

Visual observations are made (Fig 1) by introduction of aluminium paint pigment into the test fluid. With external lighting, the flake-like particles are orientated by the vortex flow so that they reflect the light differentially as they circulate. By illuminating a radial cross-section through a narrow axial slot, it is possible to observe details of the vortex structure. This method was introduced by Snyder[13] and developed by Jackson *et al*[8].

The visualization method revealed the existence of a clear sequence of secondary motions at sub-critical Taylor numbers. This sequence is illustrated in Fig 2. The initial sign of a secondary motion was the appearance of dark lines arising alternately from the inner and outer cylinder walls. These dark lines are due to the orientation of aluminium flakes, and are attributed to jet streams.

As higher Taylor numbers were reached, the jets grew outwards across the annulus, eventually being drawn into the circulation of alternately contra-rotating vortices. When $T/T_c=0.96$, a vortex system with trapezoidal shaped boundaries has developed. As T approaches T_c these gradually took the form of rectangular Taylor vortex cells. These observations confirmed the findings of previous workers.

Examination of the outer cylinder torque-speed and $\log C_F$–$\log T$ curves reveals a change of slope prior to the much larger change corresponding to T_c. Figs 3–5 show examples for both concentric and eccentric cylinders. These changes occur at $T \simeq 0.71\ T_c$ for the concentric case, $T \simeq 0.66\ T_c$ for $e=0.3$ on torque–speed curves, and at $T \simeq 0.55\ T_c$ for $e=0$ in $\log C_F$–$\log T$ curve. The non-linear behaviour of torque agrees with Preston's numerical analysis results[14] which are shown in Fig 6.

Conclusions

The experimental observations show that secondary flows at sub-critical Taylor numbers have an effect on both torque–speed and $\log C_F$–$\log T$ relationships for concentric and eccentric cylinders. This, together with the fact that their existence can be detected visually at Taylor numbers below the correct critical value are the most likely reasons for the discrepancies evident in much of the

Notation

C_F	Coefficient of friction, $G/\pi\rho\Omega_1^2R_1^4L$
d	Gap between cylinders when concentric, R_2-R_1
e	Eccentricity
F	Force, g
G	Total torque, Fr^*
L	Length of inner cylinder
r^*	Radius of restraint for the outer cylinder torque measurement, cm
R_1	Inner cylinder radius
R_2	Outer cylinder radius
T	Taylor number, $\dfrac{2\Omega_1^2R_1d^3}{\nu^2(R_1+R_2)}$
T_c	Critical Taylor number corresponding to the development of Taylor vortices
Ω_1	Rotational speed for inner cylinder
η	Radius ratio, R_1/R_2
ρ	Density
ν	Kinematic viscosity

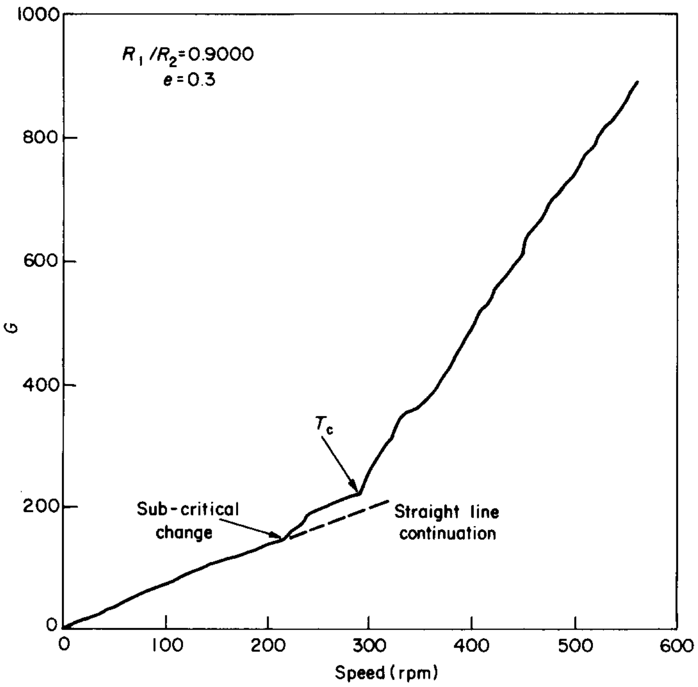

Fig 4 Effect of sub-critical vortices on torque, eccentric case

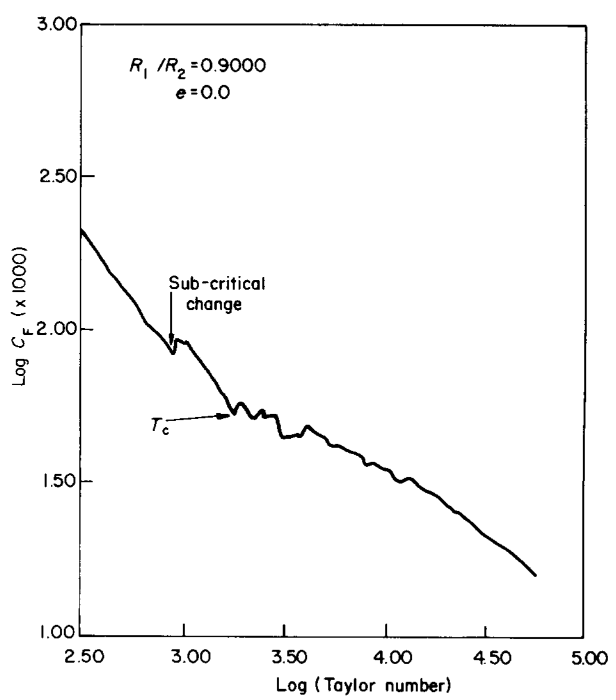

Fig 5 Effect of sub-critical vortices on C_F

published material relating to the determination of the T_c–e relationship. Failure, by some workers, to detect secondary flows visually at very low Taylor numbers lies in the inefficiency of the visualization technique employed.

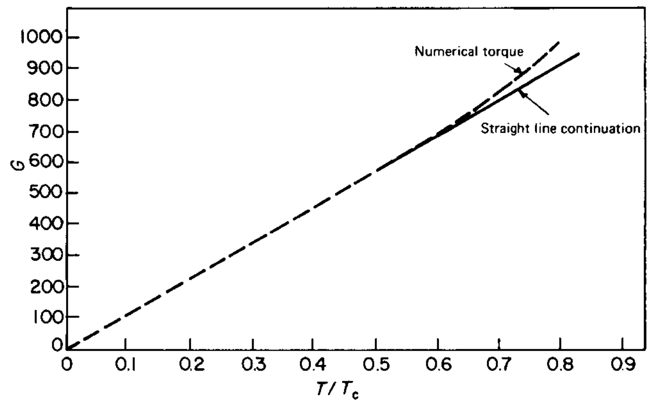

Fig 6 Preston's numerical torque $-T/T_c$ *relationship*[14] *for sub-critical region*

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